beyond ξ_M. This region appears to be related to portions of the vortex trajectory that are essentially straight. The growth of circulation due to trajectory curvature becomes insignificant under such conditions ($\kappa \to 0$) and the present model is no longer valid. In this vortex decay region a further dissipative mechanism is dominant, characterized by turbulent mixing and diffusion. Although additional measurements are needed, the stretched coordinates suggested here appear to apply in this region and the following correlation is suggested:

$$\Gamma^*/\Gamma_M^* = (\xi^*/\xi_M^*)^{-0.85}, \quad \xi^* < \xi_M^* \tag{15}$$

Conclusions

In the jet/cross-flow interaction there is an extensive region of the jet development that may be characterized by a balance between vorticity-related pressure forces and the centrifugal forces associated with jet curvature. The growth of vorticity strength in the jet is modulated by the increase in jet cross section and the spread of the vortex centers along the jet. Correlations based upon empirical expressions for vortex spacing and trajectory lead to the identification of a rapid growth in vortex strength followed by a broad plateau region where further growth is balanced by spreading of the vortex centers and flattening of the vortex trajectory. The extent of these regions depends upon the jet-to-cross-flow velocity ratio and is limited by the onset of significant diffusion due to turbulent mixing. The analytical approximations used herein, and the resulting correlations, should prove useful in modeling the external flowfield induced by the jet vortex system.

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A Lower Bound for Three-Dimensional Turbulent Separation in Supersonic Flow

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KNOWLEDGE of the lower bound, or incipient condition, for turbulent boundary-layer separation is useful as a guide to the designer of aerospace systems and to the theoretician in developing analytical or numerical methods of flow prediction.

In the two-dimensional case, such as flow over a flat plate with a spanwise compression corner as shown in Fig. 1a, the pressure rise associated with incipient turbulent boundary-layer separation increases rapidly with Mach number.[1]

In the three-dimensional case of a wedge mounted normal to a flat plate, Fig. 1d, the interaction of the skewed wedge shock with a turbulent boundary layer on the plate results in incipient separation at a near-constant pressure rise virtually independent of freestream Mach number. It is described quite well by a simple correlation[2] whereby $P_i/P_1 = 1.5$ and the component of Mach number normal to the shock is constant: $M_n = 1.2$. Thus, it would appear that, despite turbulent viscous terms, the normal and transverse components of flow can be virtually decoupled insofar as incipient separation conditions are concerned. The incipient pressure rise is much lower than for the two-dimensional case.[1]

It is of interest to determine the lower bounds for turbulent boundary-layer separation for other intermediate three-dimensional configurations such as yawed wedges on flat plates as shown in Figs. 1b and 1c. In a recent investigation of turbulent boundary-layer separation on a flat plate due to compression corners at various angles of yaw to a Mach 3 flow,[3] it was noted that, for a constant pressure rise, flow that is attached for small angles of yaw (0-10 deg) separates at larger yaw angles, with the separation zone increasing with increasing yaw angle. The authors note that "this increase agrees qualitatively with the assumption of two-dimensional flow normal to the swept corner."

This general view is consistent with the correlation of Ref. 2. Together they suggest that, in the plane of the flat plate, the component of Mach number normal to a skewed shock interacting with a turbulent boundary layer on the plate surface is far more dominant in determining the lower bound of separation or, indeed, separation characteristics themselves than the freestream Mach number, Reynolds number,[1] or wall temperature.[1]

Reference 2 notes that incipient turbulent boundary-layer separation for the two-dimensional case of a normal shock interaction occurs at a Mach number of about 1.3, somewhat higher but not much different from the three-dimensional incipient condition of $M_n = 1.2$ noted above. One may then reasonably expect that, for such three-dimensional cases as yawed wedges on flat plates, flow characteristics in a plane

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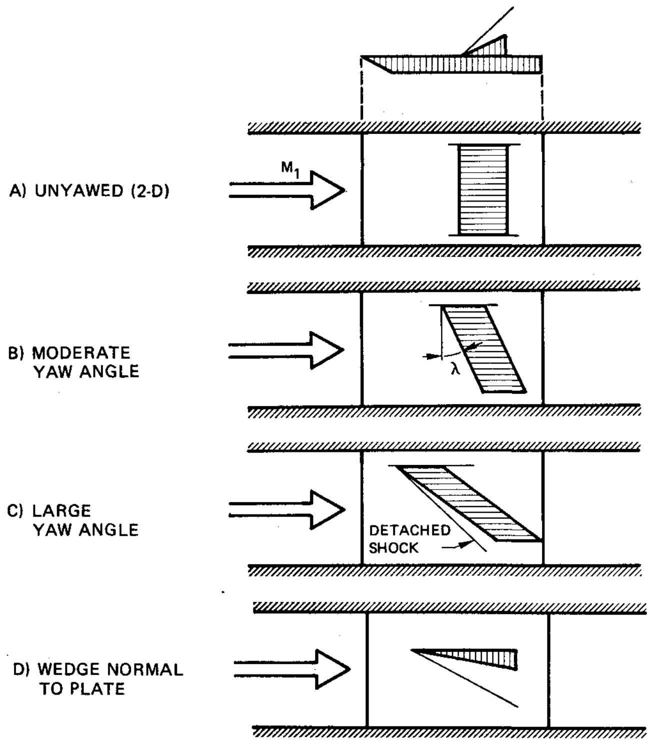

Fig. 1 Supersonic flow over wedge compression corner for different angles of yaw.

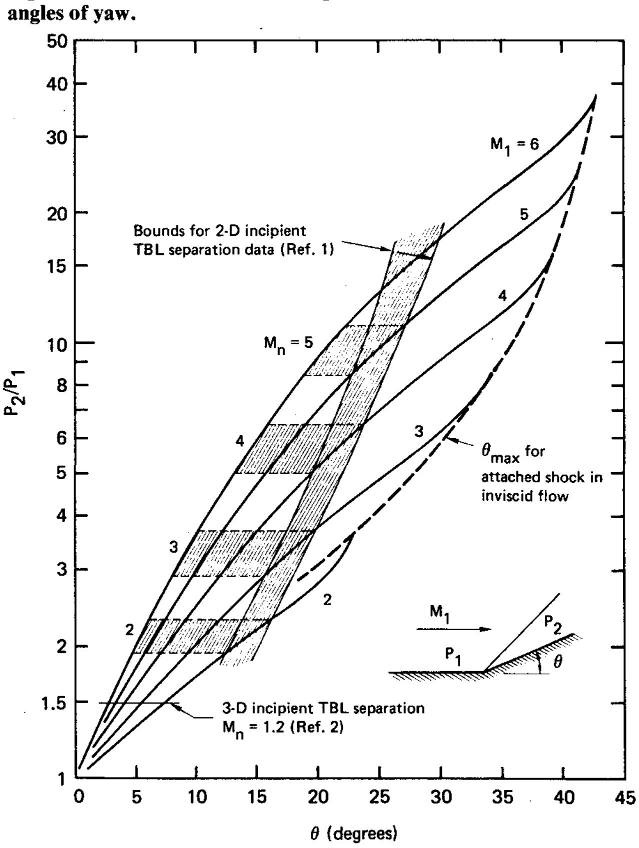

Fig. 2 Possible relationship of pressure rise for incipient turbulent separation with normal component of Mach number to yawed wedges.

normal to the shock (or wedge leading edge) may bear strong similarity to those for the two-dimensional case. Specifically, one may expect the pressure rise corresponding to the lower bound of turbulent boundary-layer separation for a yawed wedge not to be significantly different from the two-dimensional case for M_n(3-D)$=M_l$(2-D). Possible trends based on this conjecture and two-dimensional data from Ref. 1 are shown in Figs. 2 and 3 as a qualitative guide to further investigations. Figure 3 includes unpublished incipient separation data points of Settles at $M_l=2.95$ obtained at the Gas Dynamics Laboratory, Princeton University. The data points fall somewhat below the correlation for $M_l=3$, but tend to confirm the general trend.

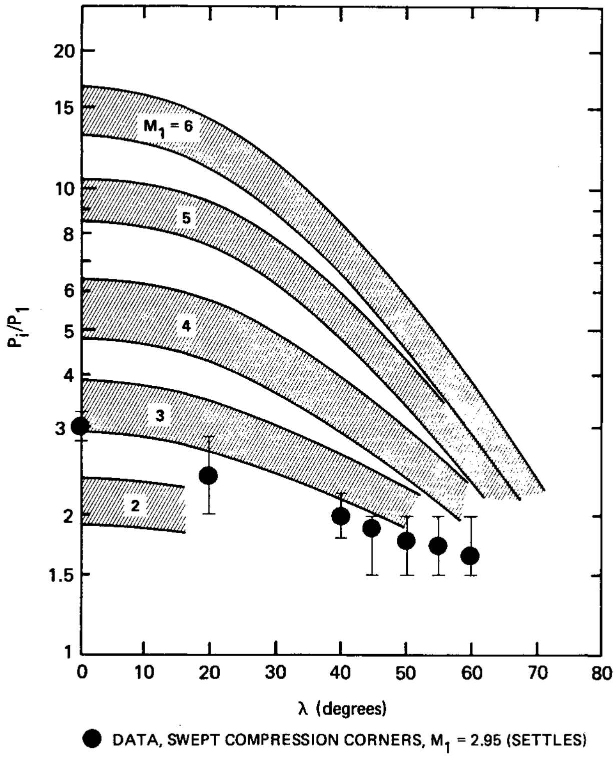

Fig. 3 Possible pressure rise for incipient turbulent separation as function of wedge yaw angle (interpreted from two-dimensional data of Ref. 1).

In Fig. 2 θ is the wedge angle measured in the streamwise direction and $M_n=M_l\cos\lambda$, where λ is the angle of yaw of the interaction line—wedge for an attached shock, or shock if detached from the wedge (see Figs. 1b and 1c). For the attached shock case the relationship between λ, θ, and the wedge angle in a plane normal to its leading edge, θ_n, is $\tan\theta=\tan\theta_n\cos\lambda$.

Thus, for example, on a flat plate at $M_l=5$ an unyawed 22 deg wedge resulting in a pressure rise by a factor of 8 is unlikely to cause turbulent boundary-layer separation. However, if this same wedge is yawed 50 deg, even though the compression angle in the freestream direction reduces to 14.5 deg and the corresponding pressure rise is reduced to about 4.5 (following down the $M_l=5$ line in Fig. 2), it may be more than sufficient to cause turbulent boundary-layer separation as $M_n=3.2$.

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